On the Importance of Humanizing Math Communication

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When I first heard about the Steven H. Strogatz Prize for Math Communication from my math teacher, I admittedly knew relatively little about math communication. I had communicated mathematical ideas in my schoolwork, of course, but seldom had I thought about the art of math communication itself. I soon found myself noticing a disconnect between the way math is typically portrayed and the way that it is actually *done*. My curiosity surrounding this topic made figuring out what I wanted to do for MoMath's contest rather simple: rather than just creating an example of effective math communication, I would write a piece describing my thoughts on this issue and, more generally, on the importance of communicating mathematical ideas in a manner that *humanizes* them.

In the remainder of this paper, I will discuss my discoveries about the disconnect I noticed as well as some thoughts from individuals who interact with the discipline of mathematicians on a professional level. I will also address what I believe to be one of the most abstract questions related to the topic at hand: Why does mathematical communication even matter?

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While textbooks and research typically make mathematics appear to be a discipline that is devoid of mistakes and that only has room for pristine, perfect work and answers, real math is messy. When the final product has yet to be reached, pages of mathematical work are covered in missteps, dead ends, and crossed-out numbers. Among the "wrong" numbers, however, are the ones that reflect growth and new understanding. This type of scratch work acts as proof of the struggle and perseverance that went into solving a problem. When the process of reaching the answer is omitted and only the final answer is kept, the messiness that is so crucial to the discipline of mathematics is allowed to be forgotten and kept hidden.

To me, the gap between the perfect math that gets published and the messier math that allows such pristine-looking work to exist seemed substantial when I first noticed it: substantial enough to explore further. Before I began the process of writing this paper, I discussed the direction of my project with my math teacher, Mr. Chris Bolognese, with whom I shared an interest in the dichotomy between the way math is so often perceived and presented and the way math truly looks most of the time. With Mr. Bolognese's assistance, I arranged virtual meetings with several mathematicians in the hope of discovering answers to some of my questions about the aforementioned gap.

Soon enough, I learned that I was not the only one who had noticed the disconnect. In my virtual conversation with Dr. Brendan Sullivan, who teaches mathematics at Emmanuel College, Dr. Sullivan revealed that while he appreciates the fact that a textbook must serve as a reference for students, he disapproves of the way in which many textbooks portray mathematics, for they make students believe that their own mathematical work should appear as pristine as the math within their textbooks. In reality, mistakes are a natural part of the process when doing mathematical work.

I later learned that research mathematician James Tanton has a similar view. When I spoke with Dr. Tanton, he explained that the papers of mathematicians in the realm of research always omit the missteps and keep only the pristine answer. This can be problematic, as it leaves the organic process of problem-solving out of the work that is published. This also contributes to the perception that, simply put, math is hard; leaving out the errors makes people believe that missteps must be anomalies and that something must be wrong with them if they make mistakes when doing math. According to Dr. Tanton, some formal math writing ends up making people feel like they cannot do math, which is a disservice to such people. Dr. Tanton also mentioned that math is often presented in a way that makes it seem to be devoid of humanity; this, he said, is the opposite effect of what math communication should have, which is to keep the humanity of math intact.

The fact that so many of my thoughts on the gap and math communication as a whole had been considered by professional mathematicians as well brought me a great deal of excitement during my virtual conversations. However, I was still curious about something else: How can we bridge the gap? Partway through each of my virtual meetings, I described a certain type of assignment that Mr. Bolognese gives his students for every unit in his math courses: the problem set. For these assignments, students must write about the process of completing a set of problems. Mr. Bolognese encourages students to include their mistakes and revisions in their problem sets, helping them to focus on their growth and learning. I have included an excerpt from one of my older problem sets in which missteps and revisions are clearly shown. In this work, it is apparent that I got stuck and reached out for help during the process of solving the problem (note that I am addressing Mr. Bolognese when using the word "you" in the pages below), and it is evident that my work is nonlinear and, to put it simply, anything but pristine.

9. If $\arccos x + \arccos y = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$.

manipulation of equation

$$\arccos x + \arccos y - \frac{\pi}{2} = 0$$

rewriting

$$\cos^{-1}x + \cos^{-1}y - \frac{\pi}{2} = 0$$

dead end (of sorts)

$$\otimes$$

At this point, I came to you for help. We made use of the following method:

manipulation of equation

$$\frac{\pi}{2} - \arccos y = \arccos x$$

$u = \arccos y$
$z = \arccos x$

substitution

$$\frac{\pi}{2} - u = z$$

note that you have multiple options here; you could use another trig function on both sides of the equation

take the sine of both sides

$$\sin\left(\frac{\pi}{2} - u\right) = \sin(z)$$

cofunction

$$\cos(u) = \sin(z)$$

"un"-substitution

$$\cos(\arccos y) = \sin(\arccos x)$$

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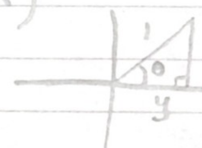
$\cos(\arccos y)$

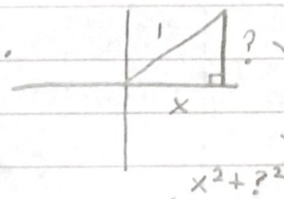


use of graph to find equivalent value

$$y = \sin(\arccos x)$$

Let's continue with this method. Before we can continue with what we have above, though, let's take a look at another part so we can incorporate some new information into what we're working on.

Looking at the RHS, sin(arccos x), what can we find? See to the right!



$x^2 + ?^2 = 1^2$

$\downarrow$

$?^2 = 1 - x^2$

$\downarrow$

$? = \sqrt{1 - x^2}$

this is the same as $x^2+y^2=1$

So: if $x^2+?^2=1$ is the same as $x^2+y^2=1$, then $?=y$

let's plug some things in, starting with $x^2+y^2=1$

$\downarrow$

$x^2 + \left(\sqrt{1-x^2}\right) = 1$

$x^2 + 1 - x^2 = 1$

$1 = 1$ ✓

so we're correct, but where do we go from here?

*

* So: with the following information...

1 ★ $\cos(\arccos y) = y$

2 ★ $y = \sqrt{1 - x^2}$

3 ★ $\cos(\arccos y) = \sin(\arccos x)$

(what we want) 4 ★ $x^2 + y^2 = 1$

...we can do the following, starting with what we had prior to this offshoot of sorts:

start	$y = \sin(\arccos x)$
	$\downarrow$
3	$y = \cos(\arccos y)$
	$\downarrow$
1	$y = y$
	$\downarrow$
2	$y = \sqrt{1 - x^2}$
	$\downarrow$
manipulation of equation (square)	$y^2 = 1 - x^2$
	$\downarrow$
manipulation of equation	$x^2 + y^2 = 1$

We've gotten to 4 (from above) — we did it! Starting with $\arccos x + \arccos y = \frac{\pi}{2}$, we've shown that $x^2 + y^2 = 1$.

Let's do this again, but this time, let's take the cosine of both sides at stage 3 and confirm that that works.

manipulation of equation

$$\frac{\pi}{2} - \arccos y = \arccos x$$

$u = \arccos y$
$z = \arccos x$

$\downarrow$

substitution

$$\frac{\pi}{2} - u = z$$

$\downarrow$

take the cosine of both sides

$$\cos\left(\frac{\pi}{2} - u\right) = \cos(z)$$

$\downarrow$

co function

$$\sin(u) = \cos(z)$$

$\downarrow$

"un"- substitution

$$\sin(\arccos y) = \cos(\arccos x)$$

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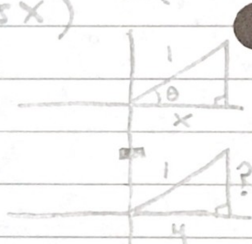
$\cos(\arccos x)$



$\downarrow$

use of graph to find equivalent value

$$\sin(\arccos y) = x$$

$\downarrow$

use of graph to find equivalent value

$$x = x$$

$\downarrow$

substitution (using work to the right)

$$x = \sqrt{1-y^2}$$

$x = ?$ $\rightarrow$ $\sin(\arccos y)$

$y^2 + ?^2 = 1^2$

$?^2 = 1 - y^2$

$? = \sqrt{1-y^2}$

$\downarrow$

$x = \sqrt{1-y^2}$

$\downarrow$

manipulation of equation (square)

$$x^2 = 1 - y^2$$

$\downarrow$

manipulation of equation

$$x^2 + y^2 = 1$$

it's what we wanted!

✓

In my view, problem sets are perhaps the most valuable assignments that Mr. Bolognese's students complete, for they enable students to practice communicating math without having to remove the humanity from their work. In fact, I would argue that the style Mr. Bolognese encourages his students to use when writing about math problems from problem sets is one that has a place in the world of published mathematical writing, as it allows the work to retain the humanity that went into it. Math is not glossy or perfect, so why should formal mathematical writing have to appear so sterile?

When I described problem sets to the mathematicians I spoke with, they responded extremely positively. Dr. Hortensia Soto, a mathematics professor at Colorado State University, explained that Mr. Bolognese's problem sets prompt his students to engage in metacognition, which is hugely beneficial when it comes to learning. Dr. Tanton mentioned metacognition as well. Dr. Tanton also made a connection between problem sets and a paper that Euler wrote that emphasized false leads, adding that including mistakes in mathematical work can be useful and can inspire new thought. Meanwhile, Dr. Sullivan discussed a textbook that he wrote in which he offers examples of what does not work and provides an explanation of how those potential missteps can lead to learning and the correct solution. According to Dr. Sullivan, incorporating mistakes in textbooks may benefit students.

While the perfect math of textbooks and research papers certainly has its place, it would be remiss not to recognize that humanized mathematical writing has value as well. By publishing work that acknowledges the human process of solving problems, math can be portrayed in a more positive light. This would help to remove the fear of making mistakes from students' minds, and it would make the discipline of mathematics seem more accessible in general. There truly is much to gain from reading and writing explanations of math that do not hide the mistakes and messiness that are so important to the subject, so I would be overjoyed to witness a transition to a more human style of mathematical writing within the mathematical community.

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I will close by offering a few possible answers to the question I raised at the beginning of this paper: Why does mathematical communication even matter? For Dr. Sullivan, math is the study of ideas. Without effective math communication, mathematical ideas, which are essential for progress as a species in many ways, will die with the people who have them. For Dr. Tanton, math is in itself a language. A calculation is a sentence with punctuation and structure: one that conveys ideas in a beautiful way. In his conversation with me, Dr. Tanton discussed the fact that math communication is hard because a person has to find a way to put their ideas in a context that another person will be able to grasp. This, he noted, is true of writing overall, and it is exceedingly important. As for myself, I would say that math communication matters because it has the potential to make math human. By making math *human*, we preserve knowledge, reveal the discipline's beauty, make it more accessible, and so much more. As a result, I cannot imagine a more important role for mathematical communication to play.